Monte Carlo Analysis for IV&V

Outline

The kinds of problems

Why it’s often hard to be certain

How it’s done

What results look like

How IV&V can be involved

Common SC Problems

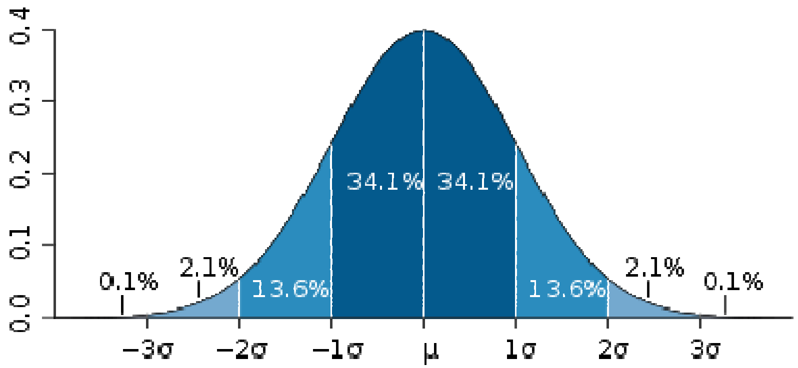
- How confident am I that a SUD will not fail?
- … not fail before time T?
- How confident booster will deliver S/C within
  - $<\sigma_x>$ meters of a specified point
  - $< \sigma_t >$ seconds of a specified time?
- How likely is a stable landing?
- Do I have enough bandwidth or other network components?

Why Not 100%

- Noise – errors – A/D – uncertainty
- Inexact measurements
- Data processing (round-off, etc.)
- Imperfect knowledge of system (& meas.)
- Not only you can't hit the nail on the head, you generally can't tell ahead of time how badly you'll miss

Standard Deviation and Confidence

- σ is a measure of spread
- $\sigma^2 = E[(x-\mu)^2]$
  - "3σ" means 99.7% for normal
- Distribution extends to ∞
- 68.2% confidence of being within σ of μ



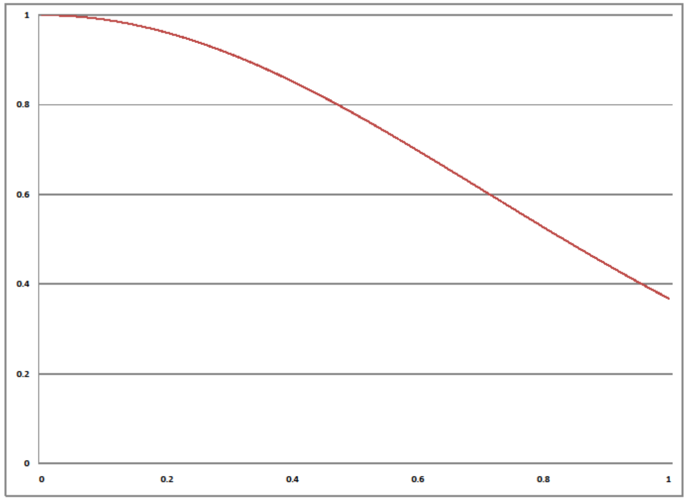
- Other distributions have μ and σ and confidence

Bathroom Scale Example

- Guaranteed accurate (big assumption!)
  - Uncertain to $\pm c$, where $c = 0.5$ or 1.0 or 0.1
  - $z = x + e$, z is measurement, x is truth, e is error
  - e has a simple probability density function
  - So we can express confidence in the reading:
    - Average error = 0
    - 100% that $|error| \leq c$
    - 50% that $|error| \leq c/2$
    - Etc
- Additional error could arise from manufacturing defects, leaving the accuracy unknown
  - Does the spring change with temperature or time (years)?
  - Does the error change with the weight? Linearly?

Fundamental Example: $I = \int_0^1 e^{-x^2} dx$

- I = fraction of unit square below curve $y = f(x) = e^{-x^2}$
- Choose N random points; $I \sim$ fraction below curve
- Better: choose N random numbers x_i
- $I \sim 1/N \sum f(x_i)$
- Increased accuracy for given N
  (smaller standard deviation)



How to do Monte Carlo Analysis

- Model or simulate process/system, errors/noise, and measurement/estimate
- Accumulate data over a set of many runs
- Compute statistics
- More sets of more runs & more statistics
- KEY QUESTION: Do the results appear to converge as you increase N?

Can/Should we do This?
(not if you can help it)

- Is there a process we can’t analyze adequately, but we can execute or simulate?
- Is “noise” complicated?
- Could our results improve a mission?

What it Looks Like

- Table showing mean & st dev vs. N to indicate convergence
- Comparison of mean & st dev for various input parameters

What's a SME to Analzye?

- Simulation for comparison with the actual process,
  - It has requirements, design, code, and test
  - Also for the measurements and estimates
- Error model
- Statistical characteristics of the input noise
- Enough runs; i.e., do you believe there is convergence?
- Do the results make sense?

Viking Lander Touchdown

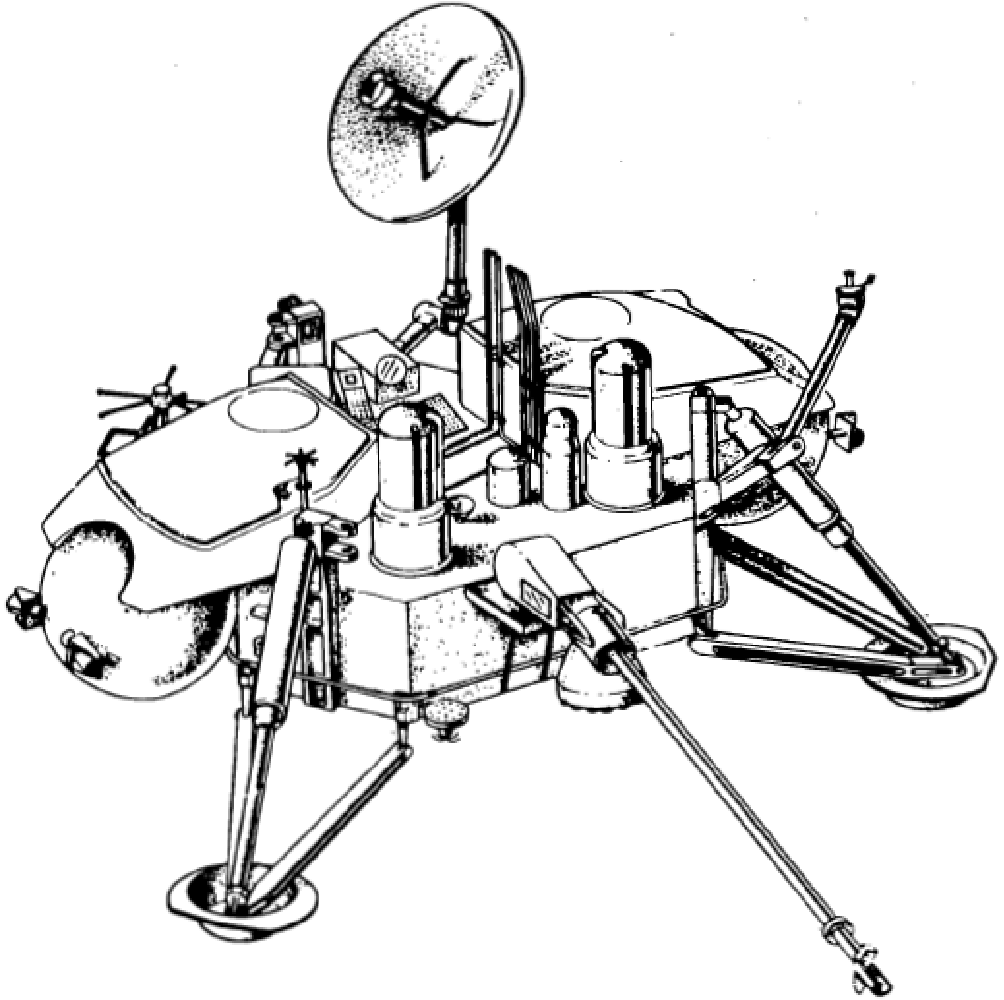


Figure 1.- Viking lander configuration.

Three Questions

1. What are the 3-sigma design values for the maximum rigid-body acceleration, minimum clearances, and maximum compression and tension strut forces and strokes?
2. What is the probability the lander will become unstable as a result of landing on a steep slope?
3. What is the probability the body of the lander will strike a rock?

Entry and Landing Phases

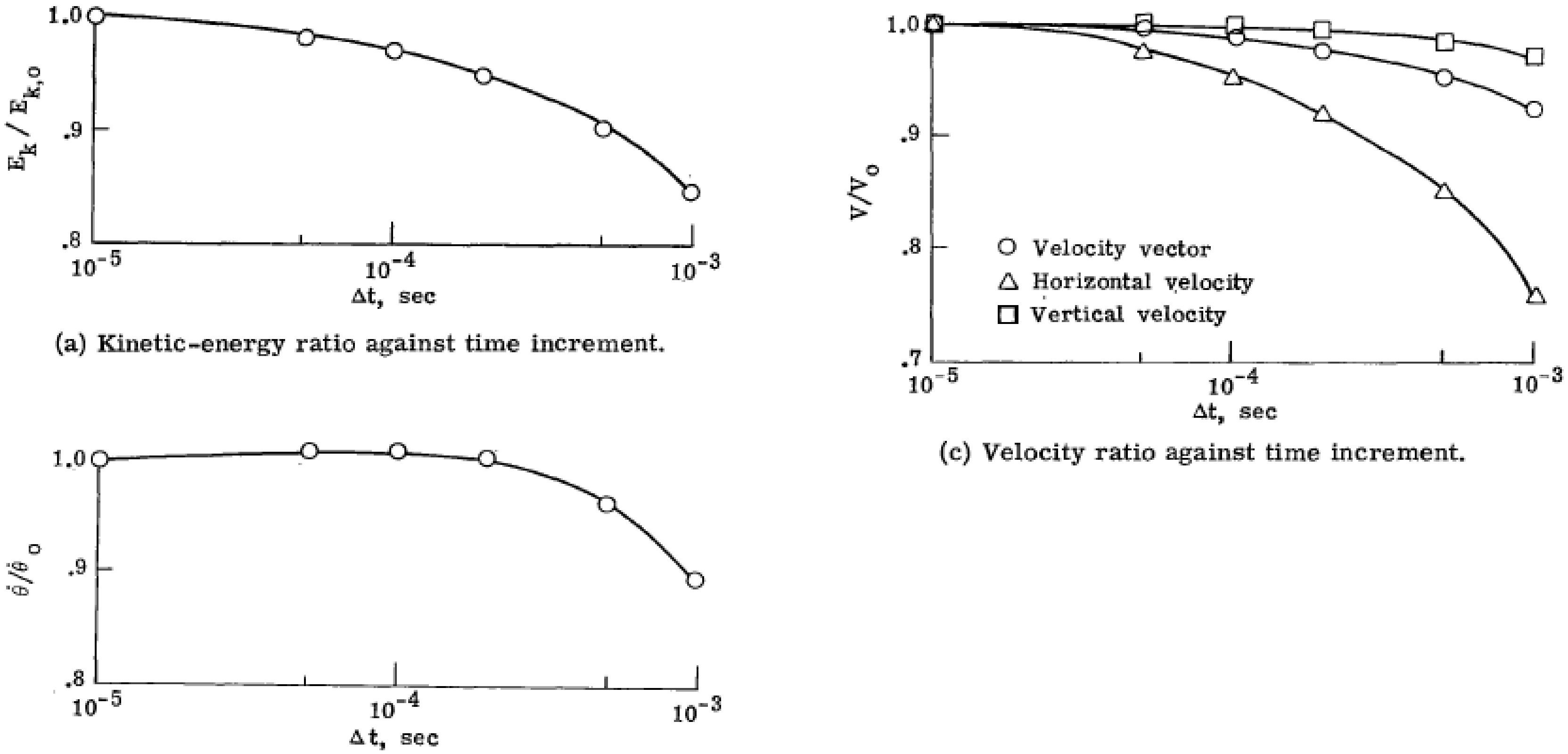
- Entry is from deorbit burn until a leg touches
  - Simulated in detail
  - Resulting mean and standard deviation used for random variables as input to next phase
- Landing is until all movement stops

Conditions at end of Entry

TABLE I.- MEANS AND STANDARD DEVIATIONS FOR INITIAL CONDITIONS OBTAINED FROM 100 TRAJECTORIES FOR THREE ATMOSPHERES

Quantity	Maximum density		Mean density		Minimum density	
	Mean	s	Mean	s	Mean	s
Engine thrust, N (lbf)						
Engine 1	756.6 (170.1)	20.5 (4.6)	756.2 (170.0)	19.6 (4.4)	754.4 (169.6)	19.6 (4.4)
Engine 2	642.8 (144.5)	20.0 (4.5)	642.8 (144.5)	19.6 (4.4)	641.9 (144.3)	19.6 (4.4)
Engine 3	751.7 (169.0)	19.1 (4.3)	751.3 (168.9)	19.1 (4.3)	750.0 (168.6)	18.7 (4.2)
Body angular rates, rad/s (deg/sec)						
Pitch	7.68×10^{-5} (0.0044)	3.26×10^{-4} (0.0187)	8.20×10^{-5} (0.0047)	3.54×10^{-4} (0.0203)	1.13×10^{-4} (0.0065)	4.03×10^{-4} (0.0231)
Yaw	3.67×10^{-5} (0.0021)	4.14×10^{-4} (0.0237)	3.67×10^{-5} (0.0021)	4.36×10^{-4} (0.0250)	3.49×10^{-5} (0.0020)	5.10×10^{-4} (0.0292)
Roll	-8.20×10^{-5} (-0.0047)	2.30×10^{-3} (0.1319)	2.97×10^{-5} (0.0017)	2.41×10^{-3} (0.1382)	-9.60×10^{-5} (-0.0055)	2.55×10^{-3} (0.1463)
Body velocity, mps (fps)						
X-axis	-0.0037 (-0.0120)	0.1042 (0.3417)	-0.0037 (-0.0122)	0.1048 (0.3437)	-0.0051 (-0.0168)	0.1063 (0.3489)
Y-axis	0.0553 (0.1815)	0.1351 (0.4434)	0.0543 (0.1781)	0.1351 (0.4432)	0.0528 (0.1731)	0.1351 (0.4434)
Z-axis	2.4257 (7.9583)	0.1320 (0.4331)	2.4322 (7.9797)	0.1316 (0.4318)	2.4388 (8.0012)	0.1313 (0.4308)

Choosing the time increment



(a) Kinetic-energy ratio against time increment.

(b) Pitch-rate ratio against time increment.

(c) Velocity ratio against time increment.